Calculation of Trim Settings for a Helicopter Rotor by an Optimized Automatic Controller

David A. Peters,* Byung S. Kim† and Han-Sheng Chen‡
Washington University, St. Louis, Missouri

An automatic feedback system, based on continuous monitoring of hub loads, is used to find the control settings that are required to obtain a given flight condition for a helicopter rotor. Optimum values of gains and time constants are determined, and the limitations of the controller are examined for flap-pitch dynamics. It is found that the present method shows good convergence and is superior to other trim techniques for systems with moderate damping or with many degrees of freedom.

Nomenclature

a	= slope of the lift curve, rad^{-1}
A_0, A_1	= collective and cyclic gains, rad^{-1}
A_i	= general gains
B	= coupling term
c	= blade chord, m
$\bar{c}$	= nondimensionalized blade chord, c/r
C_L	= hub roll moment coefficient, $L/4\pi r^2 d\rho\Omega^2 r^2$
$\bar{C}_L$	= desired roll moment, normalized, $C_L/\sigma a$
C_M	= pitch moment coefficient, $M/4\pi r^2 d\rho\Omega^2 r^2$
$\bar{C}_M$	= desired pitch moment, normalized, $C_M/\sigma a$
$[CM]$	= coupling matrix
C_T	= blade thrust coefficient, $T/4\pi r d\rho\Omega^2 r^2$
$\bar{C}_T$	= desired blade thrust, normalized, $C_T/\sigma a$
d	= length of blade section, m
e	= inertial ratio, $\sqrt{\frac{I_x}{I_y}}$
E	= inverse of error criteria
f	= ratio of errors on successive iterations
F_β	= lift force, N
$\bar{F}_\beta$	= nondimensionalized lift force, $F_\beta r/\Omega^2 I_y$
I_x, I_y	= pitch, flapping inertias, kg-m^2
J	= total number of periods of integration multiplied by degrees of freedom
K_β	= flapping spring constant
K_θ	= torsional spring constant
L	= rolling moment at hub, advancing blade down, N-m
m	= number of controls
M	= pitching moment at hub, nose up, N-m
M_θ	= moment about the pitch axis, N-m
$\bar{M}_\theta$	= nondimensionalized moment about the pitch axis, $M_\theta/I_x\Omega^2$
n	= number of second-order helicopter degrees of freedom
N	= number of periods of integration
p	= dimensionless rotating flapping frequency, $\sqrt{1+\frac{K_\beta}{\Omega^2 I_y}}$
r	= blade radius, m
t	= time, s
$^{TT}(\ )$	= cycles to convergence
T	= thrust on the blade, N
β	= flapping angle, rad
β_{pc}	= precone angle, rad
γ	= lock number, $4\rho a c d r^3/I_y$
Δ_i	= general trim errors
η	= damping of least stable mode, rad^{-1}
θ	= total pitch angle, rad
θ_i	= general control settings
θ_r	= pitch angle of root, rad, $\theta_0+\theta_s\sin\psi+\theta_c\cos\psi$
$\theta_0, \theta_c, \theta_s$	= control settings
λ	= in-flow ratio, in-flow velocity/Ωr
μ	= advance ratio, forward speed/Ωr
ρ	= density of air, kg/m^3
σ	= rotor solidity, $c/\pi r$
τ_0, τ_1	= collective and cyclic time constants, rad^{-2}
τ_i	= general time constants
ψ	= rotor azimuth angle, $\psi=\Omega t$
Ω	= angular speed of blade, rad/s
ω_θ	= dimensionless rotating pitch frequency, $\sqrt{1+\frac{K_\theta}{\Omega^2 I_x}}$
$(^*)$	$=\frac{d}{d\psi}(\ )=\frac{1}{\Omega}\frac{d}{dt}(\ )$

Introduction

PREVIOUS work in the area of helicopter-rotor stability and vibration has shown that an accurate knowledge of control settings is required in order to accurately predict blade damping or rotor loads.[1-3] In particular, fairly minor changes in control settings (1 deg) are found to result in significant changes in the stability and vibration of a helicopter. Furthermore, small deviations from the true trim condition can cause the numerical solution of the helicopter equations to diverge, with time, from the desired flight condition. This divergence is unacceptable because it may mask a true instability in the flight dynamics or in the air resonance characteristics. Until recently, however, little work has been done to develop general methods for the calculation of helicopter control settings that correspond to a given flight condition (hereafter referred to as "trim settings").

The mathematical formulation of the problem involves solution of a set of nonlinear differential equations for the periodic, equilibrium solution. This in itself may seem fairly straightforward, but the problem is complicated by the fact that there are unknown control settings (i.e., pilot inputs) that appear as forcing functions and sometimes as coefficients in the helicopter equations. These control settings must be

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*Professor and Chairman, Department of Mechanical Engineering. Associate Fellow AIAA.

†Research Assistant; presently in the Third Army of South Korea.

‡Research Assistant; presently with MTS Systems, Minnesota.

chosen in order to satisfy certain constraints on the solution. In particular, the helicopter must be flying in trim at the desired flight condition. Therefore, the analyst must apply some strategy to solve for these unknown control settings.

Several candidate solution techniques have been applied with some success, but a completely satisfactory method is yet to be determined. The most common method of solution involves an iterative procedure in which the controls are guessed and the equations are integrated in time until all transients have decayed. In general, the resultant solution will not be in trim. Therefore, a new guess of control settings must be made and a new solution found. A type of secant method is then applied to the comparative solutions in order to improve the guess on control settings iteratively until trim is obtained. The disadvantages are: 1) the large number of rotor revolutions for which the response must be calculated, and 2) the common failure of convergence near stability boundaries or at extreme flight conditions.[4] An alternate method of solution is nonlinear harmonic balance.[3] Several versions of this method are available, and they are particularly useful for the calculation of coupled rotor/body vibrations. Numerical analysis, however, indicates that other methods are often computationally more efficient than harmonic balance because of the extensive bookkeeping that is required.

A third type of solution method is periodic shooting.[5] When used for forced response (controls given), shooting is a way of iterating on initial conditions until a periodic solution is found. Thus it avoids the necessity of a wait until transients decay. In this mode, it is conceptually equivalent to the convolution method of Ref. 6. When the controls are not given, on the other hand, shooting provides that the unknown control settings can be included with the unknown initial conditions in a nonlinear programming problem. The drawbacks of shooting are an unpredictable convergence and the inefficiency for large systems. The advantages are ease of implementation and applicability even to unstable systems.

The purpose of this present work is to develop an alternative approach to trim—a feedback system (i.e., a numerical controller) that trims the helicopter automatically as the rotor equations are integrated in time. Although an analog of a pilot flying a complete rotor system would certainly be too slow to satisfy the needs of rotor trim, it may be that a very "intelligent" digital controller could quickly "fly" a single rotor blade to a desired trim condition. It is the purpose of this work to develop and test such a controller. The rotor to be trimmed will be a very simple pitch-flap model with a fixed hub. Although general rotor trim may involve four or five control variables and many degrees-of-freedom, the rotor studied here includes only three control variables and two degrees-of-freedom. Nevertheless, the fundamental elements of control are present, and the method studied is easily extended to general problems of trim.

Modeling the Rotor

The physical model used here consists of a single section of a slender, rigid, inelastic blade, which is hinged in the torsional and out-of-plane directions at the center of rotation with restraints K_θ and K_β (Fig. 1). The blade is assumed to flap with angle β, and to pitch with angle θ. The flapping spring is restrained at the root end at a precone angle β_{pc}, but the torsional spring is attached to a base point which is given a time-varying pitch schedule θ_r. The purpose of trim is to find the pitch schedule that will give a desired flight condition.

The equations of motion for the model in Fig. 1 are derived in Ref. 7. They take the nondimensional form

$$\overset{**}{\beta} + p^2\beta = (p^2 - 1)\beta_{pc} + \bar{F}_\beta \tag{1}$$

$$\overset{**}{\theta} + \omega_\theta^2\theta = (\omega_\theta^2 - 1)\theta_r + \bar{M}_\theta \tag{2}$$

where $\bar{F}_\beta$ is the nondimensional section lift, $\bar{M}_\theta$ the nondimensional section pitching moment, β_{pc} the precone angle, and θ_r the prescribed root pitch (written in terms of pilot control settings).

$$\theta_r = \theta_0 + \theta_s \sin\psi + \theta_c \cos\psi \tag{3}$$

Although a simple static stall is included in Ref. 7, here we treat only the unstalled case. Quasisteady aerodynamics without the apparent mass terms gives the following blade loads.[8]

$$\bar{F}_\beta = \frac{\gamma}{8}(1+\mu\sin\psi)\left[(1+\mu\sin\psi)\theta - \overset{*}{\beta} + \lambda + \mu\beta\cos\psi) + \frac{\bar{c}}{4}(\overset{*}{\theta}+\beta)\right] \tag{4}$$

$$\bar{M}_\theta = \frac{-\gamma}{128}\frac{\bar{c}^2}{e^2}(\overset{*}{\theta}+\beta) \tag{5}$$

where μ and λ are the nondimensional horizonal and vertical components of wind speed, and e^2 is a ratio of inertias. For the record, we note that there is an internal inconsistency implicit in these equations. In particular, Eqs. (1) and (2) imply that θ is taken about the one-quarter chord; whereas, strictly speaking, Eqs. (4) and (5) imply that θ is taken about the semichord. For a flap-torsion stability analysis, this would be unacceptable. For our purposes, however, there is little effect on the trimming problem.

To complete the rotor equations, we require a relation for the induced flow velocity, λ. For this, we use a simple, momentum-theory relationship

$$\lambda = \frac{1}{\sqrt{2}}\left[(\mu^4 + C_T^2)^{1/2} - \mu^2\right]^{1/2} \tag{6}$$

where C_T is the nondimensional rotor lift. Because we are using only a single element, this lift is linearly related to the coning angle, β.

$$C_T = \frac{\sigma a}{\gamma} p^2 \beta \tag{7}$$

We have neglected the inertial contribution $\ddot{\beta}$, because momentum theory implicitly calls for the steady (or average) value of C_T over one rotor revolution.

Similar formulas can be obtained for the steady (or average) roll and pitch moments on the rotor hub (C_L and C_M).

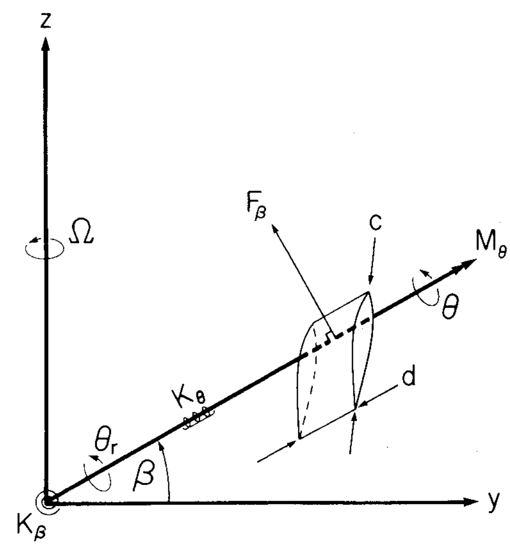

Fig. 1 Physical model of rotor blade.

$$C_L = -\frac{\sigma a}{\gamma}(p^2-1)\beta\sin\psi \qquad (8)$$

$$C_M = -\frac{\sigma a}{\gamma}(p^2-1)\beta\cos\psi \qquad (9)$$

Thus, Eqs. (7)-(9) give the hub loads in terms of the flapping angle. Simply stated, the trim problem, as defined here, is to find the control settings [$\theta_0, \theta_s, \theta_c$ in Eq. (3)] that will give desired average values of hub loads [C_T, C_L, C_M in Eqs. (7)-(9)].

Automatic Controller

The next step in the analysis is to devise a set of controller equations that will bring the control settings and hub loads to their appropriate values. We begin with a general type of controller that could be used in an arbitrary helicopter trim problem. First let us consider m control variables, θ_i, and m constraint equations formulated in standard notation as

$$\Delta_i(\theta_j) = 0 \qquad i,j = 1,m \qquad (10)$$

We define a family of automatic controllers such that they produce a rate of change of the controls that is proportional to the constraint errors, except for a first-order time lag

$$[\ \tau_i\ ]\{\ddot{\theta}_i\} + \{\dot{\theta}_i\} = [\ A_i\ ][CM]\{\Delta_i\} \qquad (11)$$

Some explanation of Eq. (11) is in order.

First, we notice the grouping $[CM]\{\Delta\}$. The matrix $[CM]$ is a coupling matrix which defines (from some a priori knowledge) what changes in θ_i one would expect to be necessary in order to eliminate the Δ_i's (i.e., drive them to zero). This matrix need not be exact, but it must generally drive θ_i in the correct direction. Equation (11) then sets the *rate of change* of each θ_i equal to some gain A_i multiplied by the predicted total change required in θ_i. For example, if the predicted change were 3 deg and if A_i were 2.0 s^{-1}, then θ_i would begin to change at 6 deg/s, and one might expect Δ_i to reach zero in 0.5 s. In reality, however, Δ_i changes continuously with θ_i so that a more likely occurrence would be an exponential decay in Δ_i with a time constant of 0.5 s.

Second, we notice that a higher order term, $\tau_i\ddot{\theta}_i$, is included in Eq. (11). This term is, in essence, a crude filter designed to suppress high-frequency oscillations in Δ_i. In other words, we wish to force only the average values of Δ_i to zero (based on the average over an entire rotor revolution). Higher-frequency oscillations in Δ_i (about zero) do not concern us. We do not desire to eliminate them nor do we want θ_i to "follow" them. Thus, the extra $\ddot{\theta}_i$ terms have been included in the equations as a filter.

We are now ready to apply Eq. (11) to our simplified rotor model. In our case, the controls are θ_0, θ_s, and θ_c. The three trim criteria are the differences between the desired hub forces and the actual, instantanteous hub forces.

$$\Delta_1 = \bar{C}_T - C_T/\sigma a = \bar{C}_T - \frac{p^2}{\gamma}\beta \qquad (12)$$

$$\Delta_2 = \bar{C}_M - C_M/\sigma a = \bar{C}_M + \frac{p^2-1}{\gamma}\beta\cos\psi \qquad (13)$$

$$\Delta_3 = \bar{C}_L - C_L/\sigma a = \bar{C}_L + \frac{p^2-1}{\gamma}\beta\sin\psi \qquad (14)$$

(We note here that β, $\beta\cos\psi$, and $\beta\sin\psi$ also represent coning and tilting of the tip path plane. Thus, one could also formulate the trim problem in terms of angles rather than forces and moments.)

The coupling matrix, $[CM]$, can be chosen based on the linear solution of Eqs. (1)-(5) with $\mu = 0$ (hover).

$$[CM] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{8(p^2-1)}{\gamma} \\ 0 & -\frac{8(p^2-1)}{\gamma} & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -B \\ 0 & -B & -1 \end{bmatrix} \qquad (15)$$

The off-diagonal terms in Eq. (15) indicate the pitch-roll coupling of the rotor in hover. The above control matrix has been found to work very well for $\mu \leq 0.5$. However, when the off-diagonal terms, B, are neglected, the controller works very poorly and often does not converge.

Finally, to complete the controller, we choose to set the gains (and time constants) in pitch and roll to be equal ($\tau_s = \tau_c = \tau_1$), ($A_s = A_c = A_1$). This is reasonable due to the symmetry of pitch and roll, and it reduces from 6 to 4 the number of free parameters in the automatic controller. Thus, the final control equations are given as follows.

$$\tau_0\ddot{\theta}_0 + \dot{\theta}_0 = A_0\Delta_1 \qquad (16)$$

$$\tau_1\ddot{\theta}_s + \dot{\theta}_s = A_1(\Delta_2 - B\Delta_3) \qquad (17)$$

$$\tau_1\ddot{\theta}_c + \dot{\theta}_c = A_1(-\Delta_3 - B\Delta_2) \qquad (18)$$

The purpose of this paper is to find the values of τ_0, τ_1, A_0, A_1 which will cause Δ_1, Δ_2, Δ_3 to be driven to zero as quickly as possible. The work presented here is based both on the results presented in Ref. 9, in which these parameters are optimized graphically, and on the extension of those results in Ref. 10, in which a more formal optimization is applied.

Numerical Results

The following section will provide a detailed study of the auto-pilot behavior under various flight conditions. Unless otherwise noted, blade parameters are as follows:

$\mu = 0.3$	$\bar{C}_L = 0.0$
$\gamma = 5.0$	$\bar{C}_M = 0.0$
$e = 0.05$	$\beta_{pc} = 0.0$
$\bar{c} = 0.1$	$\omega_\theta = 3.5$
$\bar{C}_T = 0.0159$	$p = 1.12$

The first step in the analysis is to investigate the character of the results for a reasonable set of control parameters (τ_0, τ_1, A_0, A_1). Figure 2 shows a typical response with the parameters $A_0 = 0.4$, $A_1 = 0.8$, and $\tau_0 = \tau_1 = 6\pi$. The initial conditions are set equal to zero. The resultant controls θ_0, θ_s, and θ_c (in degrees) are plotted against the number of revolutions of the rotor. The plot is generated by a predictor-corrector method that numerically integrates the equations. A single time step, adjusted internally for a desired accuracy, is used. In principle, one might want to consider a larger time step for θ_0, θ_s, θ_c and a smaller time step for β, θ, since the former vary more slowly than the latter. Here, however, we use only a single step size for all variables.

The results in Fig. 2 indicate that θ_0 seems to be underdamped, θ_c is nearly critically damped, and θ_s shows significant overdamping, but all three are converging within about 15 cycles (i.e., rotor revolutions).

A logical next step is to increase the gains ($A_0 = 1.0$, $A_1 = 1.5$) and also to lower the time constants ($\tau_0 = \pi$, $\tau_1 = 2\pi$) in an attempt to increase the convergence rate. Figures 3 and 4 show the result for the two cases: 1) zero initial conditions, and 2) initial conditions closer to the final

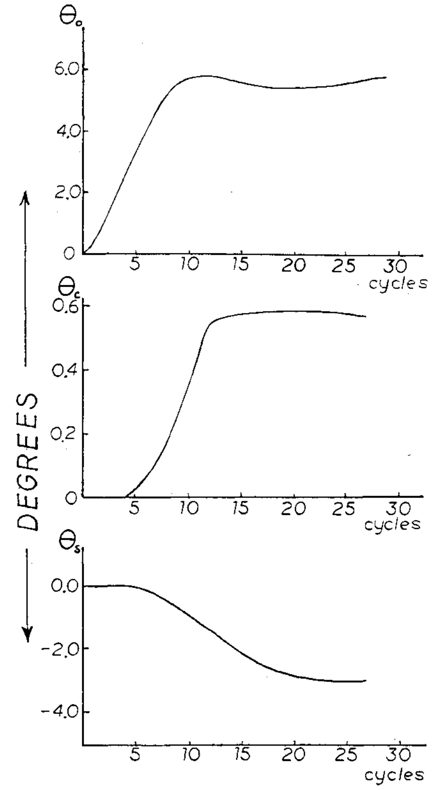

Fig. 2 Control settings vs cycles of rotation, $A_0=0.4$, $A_l=0.8$, $\tau_0=\tau_l=6\tau$.

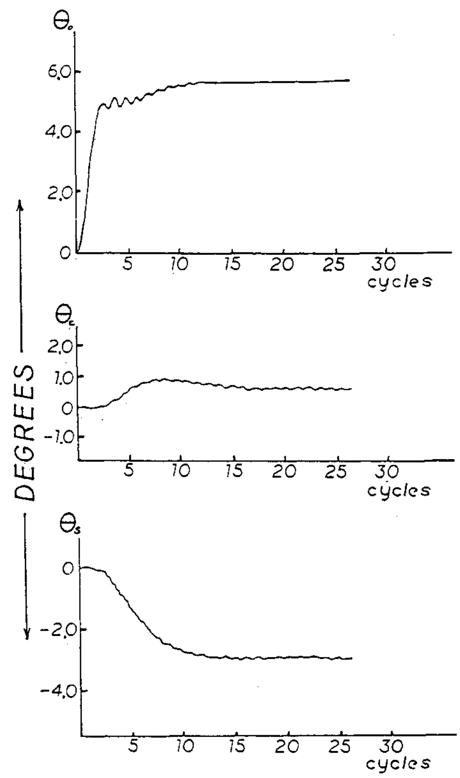

Fig. 3 Control settings vs cycles of rotation $A_0=1.0$, $A_l=1.5$, $\tau_0=\pi$, $\tau_l=2\pi$.

values. Although these results do converge faster than the previous case, they also show some oscillations about the final equilibrium. The convergence rate and final oscillations are not, however, dependent on the initial guess. The oscillations come from the response of the control system to the one- and two-per-rev hub forces. These are present because we have included only a single blade. Therefore one might suspect that if the values of the time constants are too low, these oscillations could become too large or may even become unstable. On the other hand, if the values of τ_0 and τ_l are too large, this would result in sluggish convergence. Therefore the time constant for θ_0 is temporarily fixed at $\tau_0=\pi$, and the time constant for θ_s and θ_c is chosen as $\tau_l=2\pi$.

Similarly, one would expect a limiting behavior on the gains A_0, A_l. If these gains are too small, the helicopter controls will naturally converge too slowly. On the other hand, if the gains are too large, one would expect an instability. Figure 5 provides a quantitative description of the control system behavior as a function of A_0 and A_l. The number of rotor revolutions to convergence (i.e., the number of revolutions required to obtain a periodic solution) is represented on a contour map in the A_0, A_l plane. The outermost line of the contour graph represents twenty cycles to convergence for the slowest to converge of θ_0, θ_s, or θ_c. The innermost point represents six cycles. The dashed portion of the lines represents conditions which converge, but which have oscillations of more than ± 0.5 deg in θ_0, θ_s, or θ_c, which is considered unacceptable. The system does become unstable for large gains ($A_0>2.0$, $A_l>3.0$), as expected, but the overall behavior is also more complicated than might be expected. In particular, even though the time constants are fixed ($\tau_0=\pi$, $\tau_l=2\pi$), the final oscillations turn out to be gain-dependent. This implies that there is significant coupling between the effects of the parameters. Furthermore, there seems to be a definite focus of the contours at $A_0=1.0$-1.2 and $A_l=2.0$. Thus, for the particular time constants chosen, the combination $A_0=1.2$, $A_l=2.0$ seems an excellent choice for controller gains.

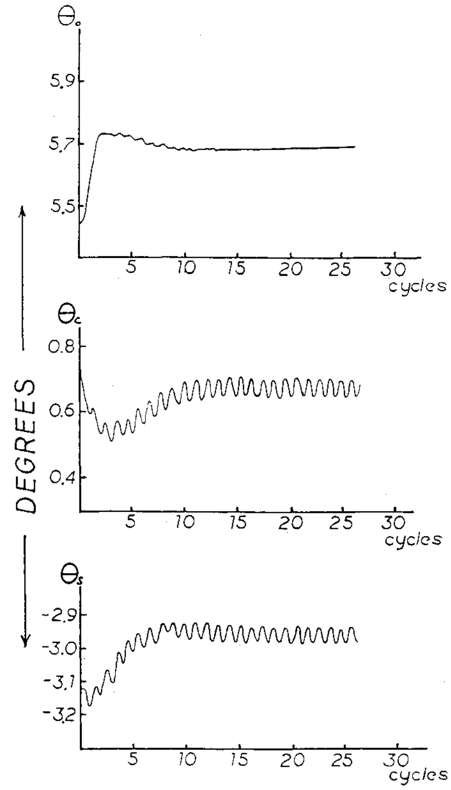

Fig. 4 Control settings for non-zero initial conditions $A_0=1.0$, $A_l=1.5$, $\tau_0=\pi$, $\tau_l=2\pi$.

Although we have found that ($A_0=1.2$, $A_l=2.0$) is the best choice for ($\tau_0=\pi$, $\tau_l=2\pi$), there is a possibility that other combinations of (τ_0,τ_l) may have different optimum (A_0, A_l) points and that the combination might be better than the one above. In order to consider this possibility, we hold the ratios $A_0/A_l=0.6$, $\tau_0/\tau_l=0.5$ (as in the above solution), but let τ_0 and A_0 vary. The results are shown in Fig. 6 in the form of contours in the A_0, τ_0 plane of constant cycles to convergence. The outermost line of the contour graph represents 12 cycles to converge (for the slowest convergence of θ_0, θ_s, or θ_c). The innermost point represents six cycles. The dashed portion of the graph corresponds to conditions which converge, but which have oscillations of more than ± 0.5 deg. The portion of the graph labeled "unstable" represents conditions for which the control settings diverge with time. The best result from Fig. 5 ($A_0 = 1.2$, $A_1 = 2.0$, $\tau_0 = \pi$, and $\tau_1 = 2\pi$) is also the best point of the curve on Fig. 6.

Therefore, within the constraints chosen, the original choice of gains and time constants still provides the best convergence. The question remains, however, as to whether even better solutions might exist outside of the constaints imposed in Figs. 4 and 5. To answer this question, a more formal optimization procedure has been applied.[10] In order to implement an optimization procedure efficiently, a function must be defined which can be minimized. We have chosen as our object function "the number of rotor revolutions required for all controls to be within ± 0.5 deg of their final values." To do this we identify the number of revolutions for convergence of each control (TT_0, TT_s, TT_c) and choose the maximum of these three as our cost function, T_{max} (Fig. 7).

Although many optimization techniques exist,[11] the particular one used here is as follows:

1) A base point is chosen and T_{max} is evaluated.

2) Local searches are made by stepping A_0 a distance 0.2 to each side (M1 direction in Table 1) and by evaluating T_{max} to see if a lower T_{max} is obtained.

3) If there is no T_{max} decrease, we do the same to M2 direction, and then to M3, M4, ..., until M42 direction, or until a decrease is found (see Table 1). We select 0.2 as increment for A_1, 0.3π fo T_0, and 0.2π for T_1.

4) If there is a T_{max} decrease, we then use the new point as a base point and go back to no. 2. If there is no decrease in T_{max}, then we assume that a local minimum point is found.

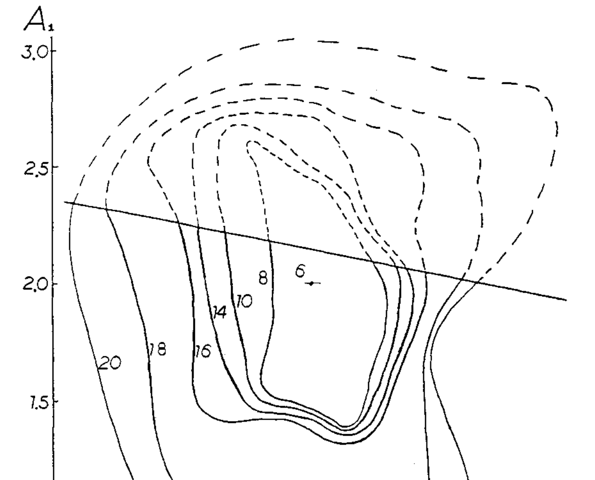

Fig. 5 Cycles to converge in A_0, A_1 plane $\tau_0 = \pi$, $\tau_1 = 2\pi$, --- implies unacceptable oscillations.

The optimum gains and time constants and the resultant controller response are given in Fig. 8. The controller has pushed θ_s and θ_c to the limiting constraint of ± 0.5 deg oscillations. If this constraint is restricted to ± 0.2 deg oscillations, a different optimum is obtained, as shown in Fig. 9. The decrease in oscillations is effected by a doubling of τ_1.

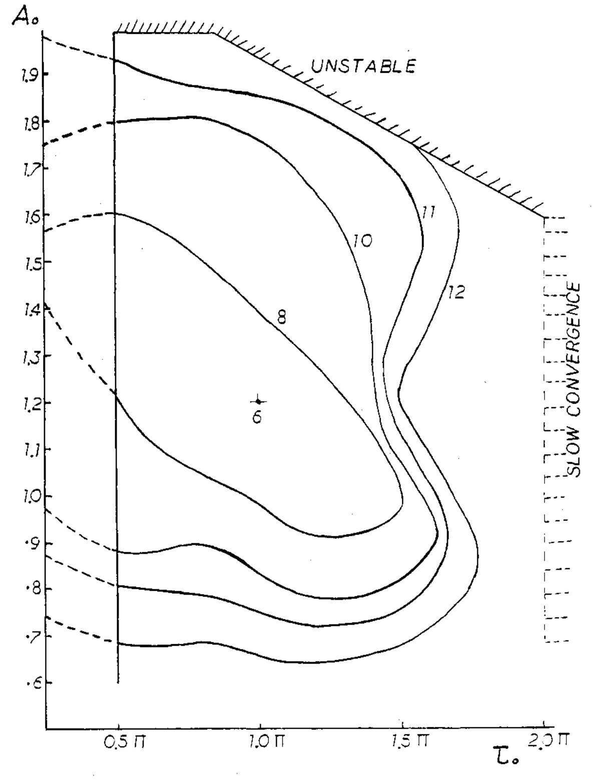

Fig. 6. Cycles to converge in A_0, τ_0 plane $A_0/A_1 = 0.6$, τ_0/τ_1, 0.5, --- implies unacceptable oscilaltions.

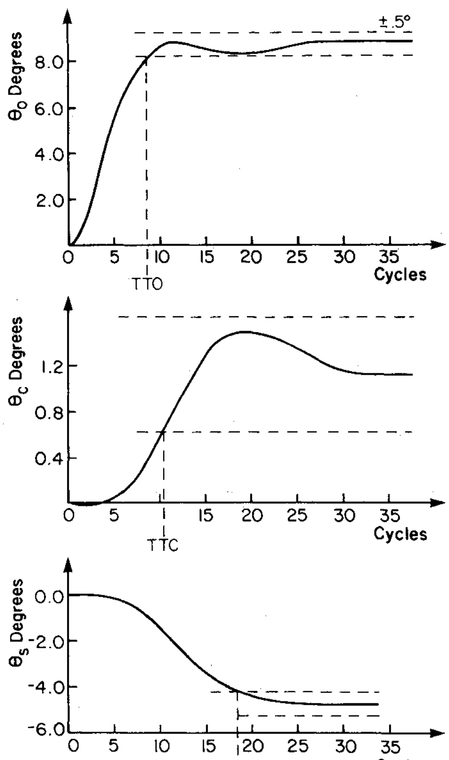

Fig. 7 Schematic of convergence times.

The larger τ_l also entails a necessary reduction in A_0 and A_l to maintain stability. The resultant penalty (for lowering ±0.5 to 0.2) is an increase of 1.28 rotor revolutions until convergence. Similar tradeoffs could be made to suit the particulars of a given problem.

More detailed results are given in Ref. 7, including the limitation of the controller with respect to torsional frequency, stall, and high advance ratio.

Comparison with Other Methods

We are now in a position to compare quantitatively the automatic controller with other methods of rotor trim. To do this, we use as a comparison the number J, which is the total number of periods of integration multiplied by the number of second-order degrees of freedom (system plus controller), $n+m$. The number of periods of integration can be computed based on the damping of the least stable mode of the system (η) and on the acceptable percentage error, $1/E$.

$$e^{-2\pi N\eta} = 1/E \qquad (19)$$

$$N = \frac{1}{2\pi\eta}\,\ell n(E) \qquad (20)$$

We must add one more period [since one does not know convergence has occurred until the $(N+1)$st revolution shows no change], and we must make sure that the controls have

Table 1 Searching directions

Direction	A_0	A_l	$T_0{}^*T_6$	$T_l{}^*T_6$	
	Increments of parameters				
M1	0.2	0.0	0.0	0.0	
M2	0.0	0.2	0.0	0.0	Change one parameter
M3	0.0	0.0	0.3	0.0	at a time.
M4	0.0	0.0	0.0	0.2	
M5	0.2	0.2	0.0	0.0	
M6	0.2	−0.2	0.0	0.0	
M7	0.2	0.0	0.3	0.0	
M8	0.2	0.0	−0.3	0.0	
M9	0.2	0.0	0.0	0.2	Change two parameters
M10	0.2	0.0	0.0	−0.2	at a time.
M11	0.0	0.2	0.3	0.0	
M12	0.0	0.2	−0.3	0.0	
M13	0.0	0.2	0.0	0.2	
M14	0.0	0.2	0.0	−0.2	
M15	0.0	0.0	0.3	0.2	
M16	0.0	0.0	0.3	−0.2	
M17	0.2	0.2	0.3	0.0	
M18	0.2	0.2	−0.3	0.0	
M19	0.2	−0.2	0.3	0.0	
M20	−0.2	0.2	0.3	0.0	
M21	0.2	0.2	0.0	0.2	
M22	0.2	0.2	0.0	−0.2	
M23	0.2	−0.2	0.0	0.2	Change three parameters
M24	−0.2	0.2	0.0	0.2	at a time.
M25	0.2	0.0	0.3	0.2	
M26	0.2	0.0	0.3	−0.2	
M27	0.2	0.0	−0.3	0.2	
M28	−0.2	0.0	0.3	0.2	
M29	0.0	0.2	0.3	0.2	
M30	0.0	0.2	0.3	−0.2	
M31	0.0	0.2	−0.3	0.2	
M32	0.0	−0.2	0.3	0.2	
M33	0.2	0.2	0.3	0.2	
M34	0.2	0.2	0.3	−0.2	
M35	0.2	0.2	−0.3	0.2	
M36	0.2	−0.2	0.3	0.2	
M37	−0.2	0.2	0.3	0.2	Change four parameters
M38	0.2	0.2	−0.3	−0.2	at a time.
M39	0.2	−0.2	0.3	−0.2	
M40	−0.2	0.2	0.3	−0.2	
M41	−0.2	0.2	−0.3	0.2	

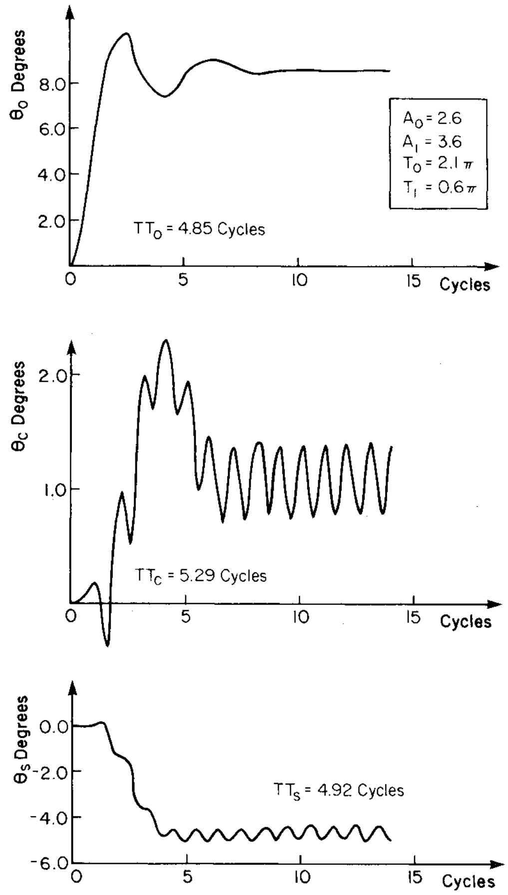

Fig. 8 Response of optimum controller based on ±0.5° oscillations.

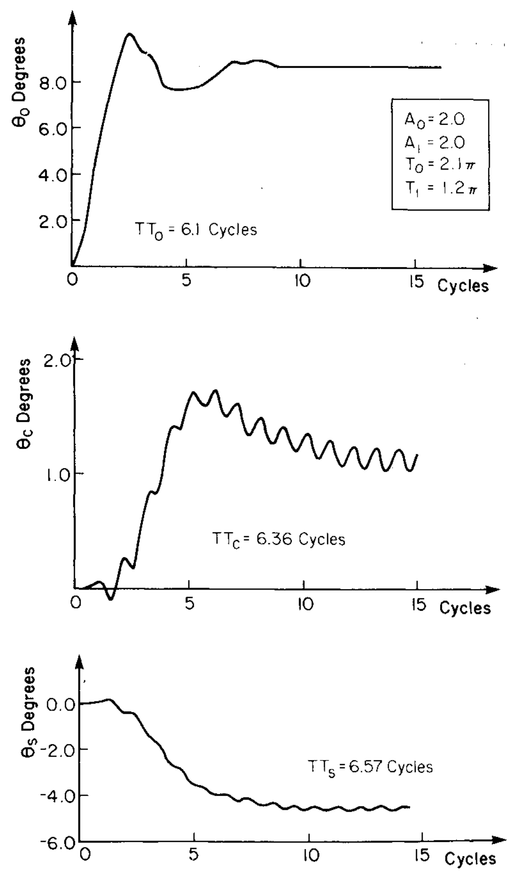

Fig. 9 Response of optimum controller based on ±0.2° oscillations.

also converged. The optimum controller takes six cycles to converge (Fig. 9). Thus, the total computing load, J, is

$$J=(n+m)\left[1+\frac{1}{2\pi\eta}\ln(E)\right] \quad \text{or} \quad J=(n+m)(6) \qquad \text{whichever is larger} \qquad (21)$$

Equation (21) simply implies that we must wait for the system *and* the controls to converge.

Similar computations can be made for periodic shooting and for the usual Newton-Raphson (or Secant) iterations.[5] These give

$$J=n\left[1+(2n+m+1)\frac{\ln(E)}{\ln(f)}\right] \qquad \text{(shooting)} \qquad (22)$$

$$J=n\left[1+\frac{\ln(E)}{2\pi\eta}\right]\left[1+(m+1)\frac{\ln(E)}{\ln(f)}\right] \qquad \text{(secant)} \qquad (23)$$

where f is the ratio of errors on successive iterations (E and f are typically ≈ 10). In order to compare the various methods, we set $E=f=10$ and equate the algebraic expressions for J from Eqs. (21)-(23). The equalities can be simplified by taking $2\pi\eta/\ln(E) \ll 1$, $m=4$, and $n+4/n+3 \approx 1$. The result is

$$\eta n > 6\frac{\ln(10)}{4\pi}=1.10 \quad \text{(secant better than shooting)} \qquad (24)$$

$$\eta n > \frac{\ln(10)}{4\pi}=0.18 \quad \text{(automatic controller better than shooting)} \qquad (25)$$

$$n \geq 3$$

Equation (25) is plotted in Fig. 10 to give an indication of the relative areas of superiority of the methods. The automatic feedback is always superior to Newton-Raphson, Eqs. (24) and (25). It is only superior to periodic shooting, however, for cases with large damping (e.g., articulated rotors) or many degrees of freedom. Thus, periodic shooting and automatic feedback together span the entire range of reasonable problems.

One disadvantage of the present controller is the necessity of fixing the matrix, $[CM]$, and of choosing the gains and time constants for each application. This preinformation is not required in periodic shooting. On the other hand, some simplified estimate of the system behavior should be found before any analysis, and $[CM]$ may not be that difficult to estimate. Furthermore, the magnitudes of the gains and time constants found here should not be much different from those of more complicated rotor systems. For example, the various gains found in this paper correspond to control rates that would bring the controls to their theoretically correct values in 1 to 3 rotor resolutions, and the time constants vary from 0.5 to 1.0 rotor revolutions. Therefore corresponding values could be used for a general rotor problem. (For rotors with b blades, time constants of $1/b$ times those here would be appropriate.)

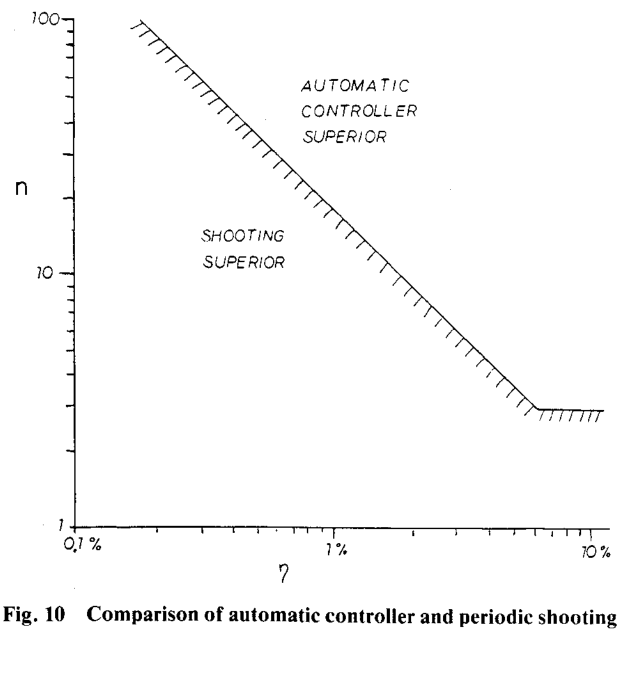

Fig. 10 Comparison of automatic controller and periodic shooting

Conclusions

The following conclusions must be tempered by the fact that we have, thus far, applied the controller to only a very simplified system.

1) An automatic controller is a viable method for obtaining rotor trim.

2) The controller can converge provided that the approximate effects of each control motion are known.

3) The gains and time constants for the best controller response are of the same order as the period of one rotor revolution.

4) The automatic controller is, in principal, superior to periodic shooting for systems with moderate damping and many degrees of freedom.

Acknowledgment

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